

# A Novel Approach to Solving the Fuzzy Assignment Problems

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## ABSTRACT

*The most successful and practical approach to solving an optimization problem is linear programming. A prime scenario of a problem is a situation where all the constraints are linear and either the function is inequalities or equivalent. Here, the problem of maximizing or minimizing an objective function was the key thing. Due to these factors, numerous types of optimization problems developed. Such as the problems with scheduling, transportation, and engineering. In the fields of logistics and business operations, for example, assignment problems are quite important. To optimally allocate a set of resources to a set of tasks is the main goal of the classical assignment problem in mathematical optimization. Here, resources should be distributed among each other so that costs are minimized. In solving such problems, the Hungarian*

*method can be pointed out as one of the most successful and widely used methods. However, problems are created with some uncertainty or ambiguity in solving them in the real world.*

*When the cost values related to the assignment problem cannot be expressed precisely, fuzzy theory is used for a range of values. It has created fuzzy assignment problems. The main purpose of these problems is to develop ways to minimize the total cost of resource distribution. There are many techniques for fuzzy set theory and fuzzy optimization in linear programming. Also, to solve the problem, researchers have proposed different methods to obtain an optimal or near-optimal solution. Thus, the purpose of this research paper is to address the above-mentioned point. For that, a new alternative method was proposed in this research paper, and its efficiency and correctness have been shown by solving numerical problems using MATLAB*

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*software.*

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## 1. INTRODUCTION

In logistics and business operations, assignment problems are extremely important, and efficient solutions for resource allocation, scheduling, and optimization are essential as well (Ghadle Kirtiwant P, 2015). There are numerous ways to explain the significance of assignment problems. They are Logistics and Supply Chain Management, Project Management, service assignment, and task allocation in a team. The main objective of the classical assignment problem in mathematical optimization is to assign a set of resources to a set of tasks in an optimal manner.

Solve the types of problems that many mathematical methods introduce in literature. Here, the popular and accurate Hungarian method can be considered one of those types of methods (Dil Afroz, 2023). However, real-life decision-makers face uncertain situations when solving optimization problems. In those special cases, some transitive values cannot be expressed clearly. Therefore, solving problems by using fuzzy values was introduced as a successful method. As a result, fuzzy assignment problems are constructed (Kumar & Gupta, 2011). Each job in the fuzzy assignment issue is linked to a group of potential resources, and each resource is linked to a group of potential tasks. Finding an assignment that generally optimizes an overall measure of compatibility or satisfaction while taking into account the degree of membership

connected to each assignment is the goal of the fuzzy assignment problems (Pal et al., 2015).

Many methods have been introduced to solve fuzzy assignment problems. It can be pointed out that they are constructed by researchers in different ways in their studies. H.W. Kuhn presented the Hungarian method to solve assignment problems related to linear programming (Kuhn, n.d.). Yuan Feng (Feng & Yang, 2006) proposed a new method considering two objectives in 2002. R. Neelambari (Neelambari et al., 2023) has recently shown in his study a new method to solve assignment problems with fuzzy numbers. In this research paper, triangular and trapezoidal problems are brought out. These papers deal with triangular fuzzy problems (Ramesh Kumar & Subramanian, 2018) and trapezoidal fuzzy problems (Arora, 2022). The main objective of this study is to propose an alternative method to obtain solutions to fuzzy assignment problems. Based on Yager's Robust's ranking method, the objective is to study the problems with triangular and trapezoidal assignment problems and carry out the optimization process through an alternative method. Also, solutions are to be obtained through MATLAB-Software, to simplify this.

## 2. METHODOLOGY

Assignment problems are a linear programming model. Also, fuzzy assignment models can be summarized in the following form:

Table 1. cost matrix of fuzzy assignment problem

In

	Job (Resource) 1	Job (Resource) 2	...	Job (Resource) m
Worker (task) 1	$\tilde{C}_{11}$ $X_{11}$	$\tilde{C}_{12}$ $X_{12}$	...	$\tilde{C}_{1n}$ $X_{1n}$
Worker (task) 2	$\tilde{C}_{21}$ $X_{21}$	$\tilde{C}_{22}$ $X_{22}$	...	$\tilde{C}_{2n}$ $X_{2n}$
Worker (task) n	$\tilde{C}_{m1}$ $X_{m1}$	$\tilde{C}_{m2}$ $X_{m2}$	...	$\tilde{C}_{mn}$ $X_{mn}$

**2.1 fuzzy assignment problem**

The fuzzy assignment problem can be represented by the following cost matrix:

In our general idea of the solution to the problem, we generalized the linear programming model (Balaji & Sundar, 2020).

$c_{ij} : [\tilde{c}_{ij}]$  is called the fuzzy cost matrix. Where  $i^{th}$  job (set of resources) to  $j^{th}$  workers (set of tasks) (E. M. U. S. B.Ekanayake, 2021)

Let's take the assignment problem with n jobs and n workers. Cost defines the assigning of workers i to job j in the following way:

$$x_{ij} = \begin{cases} 1; & \text{if worker } i \text{ assigned for job } j \\ 0; & \text{if worker } i \text{ not assigned job } j \end{cases}$$

Objective function: minimize the total cost

$$\text{Minimize } \sum_{i=1}^m \sum_{j=1}^n X_{ij} \tilde{C}_{ij}$$

Subject constraints.

One worker can be assigned one job.

$$\sum_{j=1}^n X_{ij} = 1, i = 1, 2, \dots, m$$

One job can be assigned to one worker.

$$\sum_{i=1}^m X_{ij} = b_j, j = 1, 2, \dots, n \text{ and}$$

$$X_{ij} \geq 0 \text{ for all } i = 1, 2, \dots, m, j = 1, 2, \dots, n$$

this research, we use fuzzy concept-related assignment problems.

**2.2 Fuzzy set**

To describe problems with mathematics with ambiguous and uncertain values, the fuzzy set idea was invented. The formula that determines how much of a given element is a member of a fuzzy set. Additionally, membership functions have a value in the [0, 1] range. It deals with values that have a strict boundary and are either 1 or 0, and has crisp, precise values (Dijkman et al., 1983).

The triangular fuzzy number set can be written as follows in mathematical notation (Hunwisai & Kumam, 2017):

$\tilde{a} = (a_1, a_2, a_3)$  is triangular fuzzy number with membership function is mention follows;

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & a_1 \leq x \leq a_2 \\ \frac{(a_3-x)}{(a_3-a_2)} & a_2 \leq x \leq a_3 \\ 0 & \text{else} \end{cases}$$

The trapezoidal fuzzy number set can be written as follows in mathematical notation (Hunwisai & Kumam, 2017):

$\tilde{a} = (a_1, a_2, a_3, a_4)$  is triangular fuzzy number with membership function is mention follows;

$$\mu_{\tilde{A}}(x) \begin{cases} 0 & x \leq a_1 \\ \frac{(x - a_1)}{(a_2 - a_1)} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{(a_4 - x)}{(a_4 - a_3)} & a_3 \leq x \leq a_4 \\ 0 & else \end{cases}$$

In this research, using Robust's ranking technology, ranking fuzzy numbers is a necessary step in many mathematical models. In the study, we used fuzzy assignment problems related to triangular and trapezoidal fuzzy numbers. Following is a discussion of Yager's robust ranking mathematical formula for triangular and trapezoidal fuzzy sets.

Triangular fuzzy set

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u) d\alpha$$

Where  $(a_\alpha^l, a_\alpha^u)$  is  $\alpha$  cut level of the fuzzy number  $\tilde{a}$

$$a_\alpha^l = (a_2 - a_1)\alpha + a_1$$

$$a_\alpha^u = (a_3 - a_2)\alpha + a_3$$

Trapezoidal fuzzy set

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u) d\alpha$$

Where  $(a_\alpha^l, a_\alpha^u)$  is  $\alpha$  cut level of the fuzzy number  $\tilde{a}$

$$a_\alpha^l = (a_2 - a_1)\alpha + a_1$$

$$a_\alpha^u = (a_4 - a_3)\alpha + a_4$$

### 2.3 proposed method

Step 1: First, obtain the crisp values. related fuzzy cost set using Yager's robust ranking method.

Step 2: Define the cost matrix according to the assignment problem. Workers (W1,

W2, and Wn) represent each row, and jobs (J1, J2, and Jn)

represent each column in the problem.

Step 3: After that, divide those cost values by the total cost values in the column corresponding to each other to get the ratio of the cost values relative to each other. Named the column ratio matrix.

Step 4: Then, in the same way, divide those cost values by the total cost values in the row corresponding to each

other to get the ratio of the cost values relative to each other. Named the row ratio matrix.

Step 5: After that, multiply the two corresponding ratio values of the row ratio matrix and the column ratio matrix to get a new matrix.

Step 6: Select the value in the increasing order of the ratio values, mark it, and cross that value.

in the corresponding column and row, and do the process for each remaining row and column until all the rows and columns are crossed.

Step 7: Mention the selected values in the original cost matrix in relation to the places marked as such.

Step 8: At the end, give the sum of those values as the total assignment value.

### 3. RESULT AND DISCUSSION

By using the algorithm mentioned above, the numerical example can be shown as follows: Triangular and trapezoidal fuzzy assignment problems are illustrated in it. Also, the calculations related to the first step of the algorithm are shown, and the

calculations related to all the other steps are shown in MATLAB in figures 1 and 2.

Ex.1

fuzzy assignment problem (triangular fuzzy set)

$$[\tilde{c}_{ij}] = \begin{pmatrix} (10,20,30) & (10,20,40) & (10,30,40) & (10,20,30) \\ (10,20,40) & (10,30,40) & (10,20,30) & (20,30,40) \\ (10,20,40) & (20,30,40) & (20,30,40) & (10,20,30) \\ (20,30,50) & (10,20,30) & (20,40,60) & (20,30,50) \end{pmatrix}$$

According to the step 01, obtain the crisp values of each fuzzy cost using following Yager’s robust ranking technique

$$R(\tilde{C}_{11}) = R(10,20,30)$$

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u) d\alpha$$

Where:

$$a_\alpha^l = (20-10)\alpha + 10 = 10\alpha + 10$$

$$a_\alpha^u = -(30-20)\alpha + 30 = -10\alpha + 30$$

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u) d\alpha$$

$$\int_0^1 0.5(40) d\alpha = 20$$

Similar way we can obtain the following crisp values

$$R(\tilde{C}_{11}) = 20, R(\tilde{C}_{12}) = 22.5, R(\tilde{C}_{13}) = 27.5, R(\tilde{C}_{14}) = 20$$

$$R(\tilde{C}_{21}) = 22.5, R(\tilde{C}_{22}) = 27.5, R(\tilde{C}_{23}) = 20, R(\tilde{C}_{24}) = 30$$

$$R(\tilde{C}_{31}) = 22.5, R(\tilde{C}_{32}) = 30, R(\tilde{C}_{33}) = 30, R(\tilde{C}_{34}) = 20$$

$$R(\tilde{C}_{41}) = 32.5, R(\tilde{C}_{42}) = 20, R(\tilde{C}_{43}) = 40, R(\tilde{C}_{44}) = 32.5$$

Step 2,3,4,5,6,7 &8 show the following worksheet interface in “MATLAB software”

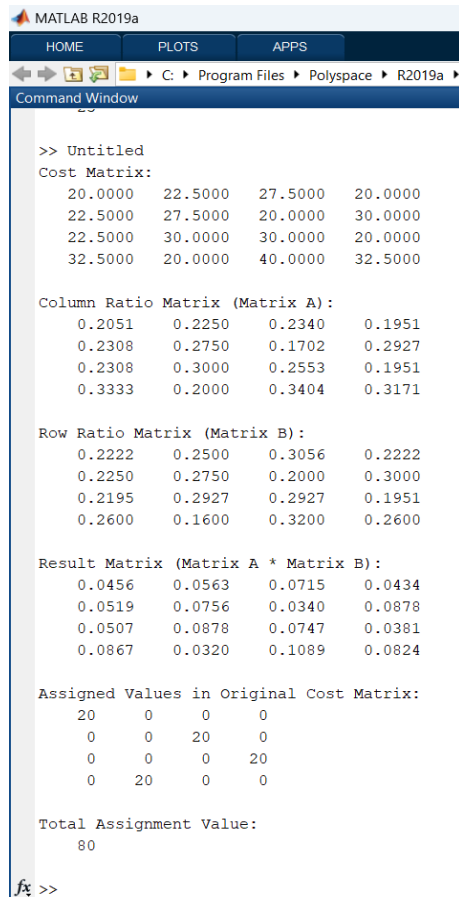


Figure 1. MATLAB worksheet for example 1

total assignment cost = 80

Ex.2

Table 01: fuzzy assignment problem (trapezoidal fuzzy set)

$$\tilde{c}_{ij} = \begin{pmatrix} (10,20,30,40) & (10,20,40,50) & (10,30,40,50) & (10,20,30,40) \\ (10,20,40,50) & (10,30,40,50) & (10,20,30,40) & (20,30,40,50) \\ (10,20,40,50) & (20,30,40,50) & (20,30,40,50) & (10,20,30,40) \\ (20,30,50,60) & (10,20,30,40) & (20,40,60,80) & (20,30,50,60) \end{pmatrix}$$

According to the step 01, obtain the crisp values of each fuzzy cost using following Yager’s robust ranking technique

$$R(\tilde{C}_{11}) = R(10,20,30,40)$$

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u) d\alpha$$

Where:

$$a_\alpha^l = (20-10)\alpha + 10 = 10\alpha + 10$$

$$a_\alpha^u = -(40-30)\alpha + 30 = -10\alpha + 40$$

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^l, a_\alpha^u) d\alpha$$

$$\int_0^1 0.5(50) d\alpha = 25$$

Similar way we can obtain the following crisp values

$$R(\tilde{C}_{11}) = 25, R(\tilde{C}_{12}) = 30, R(\tilde{C}_{13}) = 32.5, R(\tilde{C}_{14}) = 25$$

$$R(\tilde{C}_{21}) = 30, R(\tilde{C}_{22}) = 32.5, R(\tilde{C}_{23}) = 25,$$

$$R(\tilde{C}_{24}) = 25$$

$$R(\tilde{C}_{31}) = 30, R(\tilde{C}_{32}) = 35, R(\tilde{C}_{33}) = 35, R(\tilde{C}_{34}) = 25$$

$$R(\tilde{C}_{41}) = 40, R(\tilde{C}_{42}) = 25, R(\tilde{C}_{43}) = 50, R(\tilde{C}_{44}) = 40$$

Step 2,3,4,5,6,7 &8 show the following worksheet interface in “MATLAB software

Figure 2. MATLAB worksheet for example 2 total assignment cost = 100

The stepwise analysis of the problem solved

by the proposed algorithm is given above. In addition, the results obtained for several numerical problems are shown in tables and figures. It shows the results of the proposed algorithm as well as the solution values obtained by the Hungarian method using Yager's method.

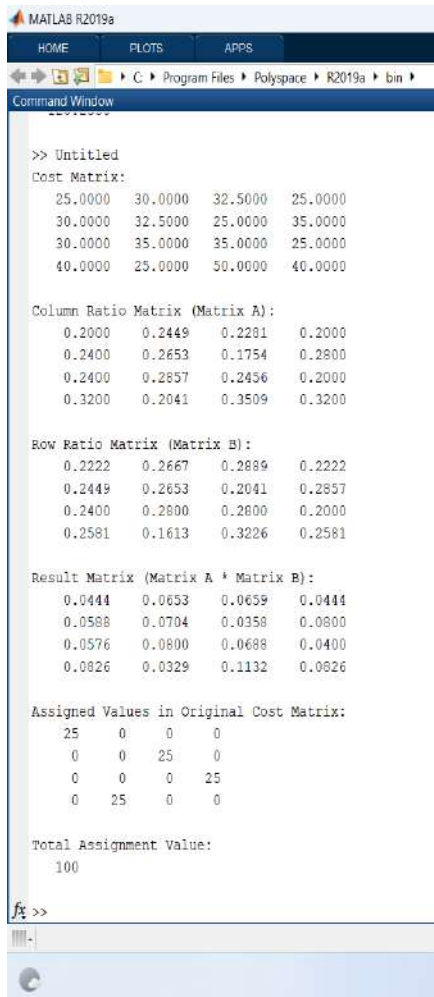


Table 2. comparison of the optimal and proposed method solutions

Ex.1 (Kadhirvel & Balamurugan , 2012)	Proposed method	80
	Optimal solution (using Hungarian method)	80
Ex.2 (Akila & Kavinelaavu, 2022)	Proposed method	100
	Optimal solution (using Hungarian method)	100
Ex.3 (Amit	Proposed	16

Kumar, 2009)	method		i, 2020)	Optimal solution (using Hungarian method)	25.25
	Optimal solution (using Hungarian method)	16			
Ex.4 (Jahirhussain & Jayaraman, 2013)	Proposed method	86.25	<p>Figure 3. comparison of the optimal and proposed method solutions</p>		
	Optimal solution (using Hungarian method)	86.25			
Ex.5 (Kalaiarasi et al., 2014)	Proposed method	23	<p>In order to accurately display the comparison based on the comparative data shown in Table No. 2 above, we applied a bar graph. Figure 3 refers to it. For each case, there are two sets of information. "Proposed Method" and "Optimal Solution (Using the Hungarian Method)" are the two alternates. Here, it is logical to display the numerical representation using a bar chart. Each example is organized into two columns: "Proposed Method" and "Optimal Solution (Using Hungarian Method)". Each column's height depends on the numerical value used in that approach.</p>		
	Optimal solution (using Hungarian method)	23			
Ex.6 (Muamer, 2020)	Proposed method	49.5	<p><b>4. CONCLUSIONS</b></p> <p>The fuzzy assignment problem is a special type of assignment problem, which is another type of optimization problem. Here, the problems in certain uncertain situations are discussed. The primary objective of these problems is to allocate a set of resources to a set of tasks in an optimal</p>		
	Optimal solution (using Hungarian method)	49.5			
Ex.7 (Muruganandam & Hema, 2019)	Proposed method	12			
	Optimal solution (using Hungarian method)	12			
Ex.8 (Thangapand	Proposed method	25.25			

manner. In relation to the problem, assignments should be made at the lowest possible cost. The fuzzy nature of assignments should be taken into account when solving fuzzy assignment problems. It should decide how the overall assignment will be done. Here, it can be pointed out that various mathematical models and algorithms, including various optimization techniques, have been used to obtain an optimal or near-optimal solution by solving fuzzy assignment problems. The primary purpose of this research paper was to introduce such an algorithm.

Most of the steps related to the algorithm we used here were solved in MATLAB," and by doing so, we tried to maximize the efficiency of the algorithm. Here, the solution of the proposed method and the optimal solution through eight examples are shown in Figure 2 and Table 1 in comparison. It was confirmed that the values of the proposed algorithm and the optimal solution are the same. However, it cannot be determined from these eight problems alone that the proposed method always gives the optimal solution. For that, it is necessary to analyse a wide range of special problems related to different situations. But looking at the overall model, it can be concluded that this model gives good solutions for fuzzy assignment problems by getting the optimal solution for these problems. The primary objective of this research paper was to propose a method that can obtain an optimal or near-optimal solution for fuzzy assignment problems. Accordingly, as a final conclusion, it was possible to introduce an efficient alternative algorithm that provides efficient solutions for fuzzy assignment problems.

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**APPENDIX**

Table 03. example of the fuzzy assignment problem

Example	$\tilde{C}_{ij}$
Ex.1	[(10,20,30),(10,20,40),(10,30,40),(10,20,30);(10,20,40),(10,30,40),(10,20,30),(20,30,40);(10,20,40),(20,30,40),(20,30,40),(10,20,30);(20,30,50),(10,20,30),(20,40,60),(20,30,50)]
Ex.2	[(10,20,30,40),(10,20,40,50),(10,30,40,50),(10,20,30,40);(10,20,40,50),(10,30,40,50),(10,20,30,40),(20,30,40,50);(10,20,40,50),(20,30,40,50),(20,30,40,50),(10,20,30,40);(20,30,50,60),(10,20,30,40),(20,40,60,80),(20,30,50,60)]
Ex.3	[(1,5,9),(8,9,10),(2,3,4);(7,8,9),(6,7,8),(6,8,10);(5,6,7),(6,10,14),(10,12,14)]
Ex.4	[(1,4,9),(16,25,36),(9,36,49);(16,25,64),(36,64,81),(4,49,64);(4,25,81),(25,36,64),(49,64,81)]
Ex.5	[(1,5,9),(3,7,11),(7,11,15),(2,6,10);(4,8,12),(1,5,9),(4,9,13),(2,6,10);(0,4,8),(3,7,11),(6,10,14),(3,7,11);(6,10,14),(0,4,8),(4,8,12),(-1,3,7)]
Ex.6	[(11,12,13),(5,6,13),(7,9,11),(3,10,11),(3,8,13),(12,14,16),(7,9,11);(7,9,11),(9,10,11),(5,8,11),(8,10,12),(5,6,7),(9,11,13),(8,10,11);(3,9,15),(7,8,13),(6,7,9),(4,5,12),(4,6,12),(8,15,18),(3,4,17);(5,6,13),(13,17,21),(5,7,13),(8,10,16),(8,15,18),(7,11,15),(3,9,13);(6,8,10),(6,9,12),(9,11,13),(11,13,15),(8,9,10),(6,8,12),(4,8,12);(10,14,18),(9,13,17),(4,6,17),(2,4,18),(3,8,17),(6,12,14),(7,8,13);4,7,10,(7,8,13),(5,6,7),(1,8,13),(8,9,14),(5,12,19),(7,8,17)]
Ex.7	[(1,2,3),(3,4,5),(5,6,7),(7,8,9);(2,3,4),(4,5,6),(7,8,9),(1,2,3);(4,5,6),(6,7,8),(2,3,4),(1,2,3);(3,4,5),(4,5,6),(5,6,7),(6,7,8)]

Ex.8 [(3,5,6,7),(5,8,11,12),(9,10,11,15),(5,8,10,11);(7,8,10,11),(3,5,6,7),(6,8,10,12),(5,8,9,10);  
(2,4,5,6),(5,7,10,11),(8,11,13,15),(4,6,7,10); (6,8,10,12),(2,5,6,7),(5,7,10,11),(2,4,5,7)]

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